1.4 Projectile motion

A projectile is any object that is thrown or projected into the air and is moving freely, i.e. it has no power source (such as a rocket engine) driving it. A netball as it is passed, a coin that is tossed and a gymnast performing a dismount are all examples of projectiles. If they are not launched vertically and if air resistance is ignored, projectiles move in *parabolic* paths.

If air resistance is ignored, the only force acting on a ***PROJECTILE*** during its flight is its weight, which is the force due to gravity, F_g or W. This force is constant and always directed vertically downwards, and causes the projectile to continually deviate from a straight line path to follow a parabolic path.

- Given that the only force acting on a projectile is the force of gravity, F_g, it follows that the projectile must have a vertical acceleration of 9.8 m s^{-2} downwards.
- The only force, F_g, that is acting on a projectile is vertical and so it has no effect on the horizontal motion. The *vertical and horizontal components of the motion are independent of each other* and must be treated separately.
- There are no horizontal forces acting on the projectile, so the horizontal component of velocity will be constant.

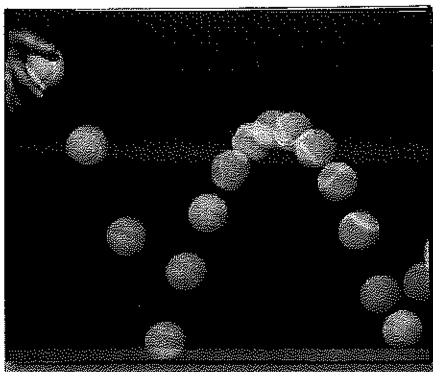


Figure 1.19 A multiflash photograph of a tennis ball that has been bounced on a hard surface. The ball moves in a parabolic path.

In the ***VERTICAL COMPONENT***, a projectile accelerates with the acceleration due to gravity, 9.8 m s^{-2} downward.
In the ***HORIZONTAL COMPONENT***, a projectile moves with uniform velocity since there are no forces acting in this direction.

These points are fundamental to an understanding of projectile motion, and can be seen by studying Figure 1.20.

PRACTICAL ACTIVITY 8
Projectile motion

If *air resistance is ignored*, the motion of a projectile will be symmetrical around the vertical axis through the top of the flight. This symmetry extends to calculations involving speed, velocity and time as well as position. As seen in Figure 1.21, the projectile will take exactly the same time to reach the top of its flight as it will to travel from the top of its flight to the ground. At any given height, the speed of the projectile will be the same, and at any given height the velocities are related. On the way up, the angle for the velocity vector will be directed above the horizontal, whereas on the way down, the angle is the same but it is directed below the horizontal. For example, a projectile launched at 50 m s^{-1} at 80° above the horizontal will land at 50 m s^{-1} at 80° below the horizontal.

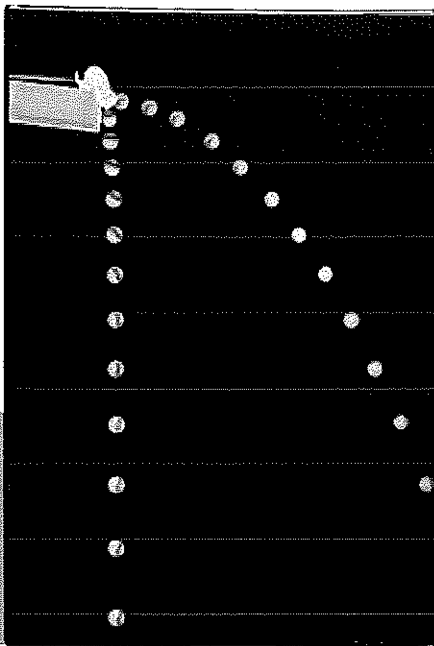


Figure 1.20 A multiflash photo of two golf balls released simultaneously. One ball was launched horizontally at 2.0 m s^{-1} while the other was released from rest. The projectile launched horizontally travels an equal horizontal distance during each flash interval, indicating that its horizontal velocity is constant. However, in the vertical direction, this projectile travels greater distances as it falls. In other words, it has a vertical acceleration. In fact, both balls have a vertical acceleration of 9.8 m s^{-2}, so they both fall at exactly the same rate and land at the same time. This shows that the two components of motion are independent: the horizontal motion of the launched projectile has no effect on its vertical motion (and vice versa).

Physics file

A common misconception is that there is a driving force acting to keep a projectile moving through the air. This is a medieval understanding of motion. Such a force does not exist. For example, when you toss a ball across the room, your hand exerts a force on the ball as it is being thrown, but this force stops acting when the ball leaves your hand. There is no driving force propelling the ball along. Only the forces of gravity and air resistance act on the ball once it is in mid-air.

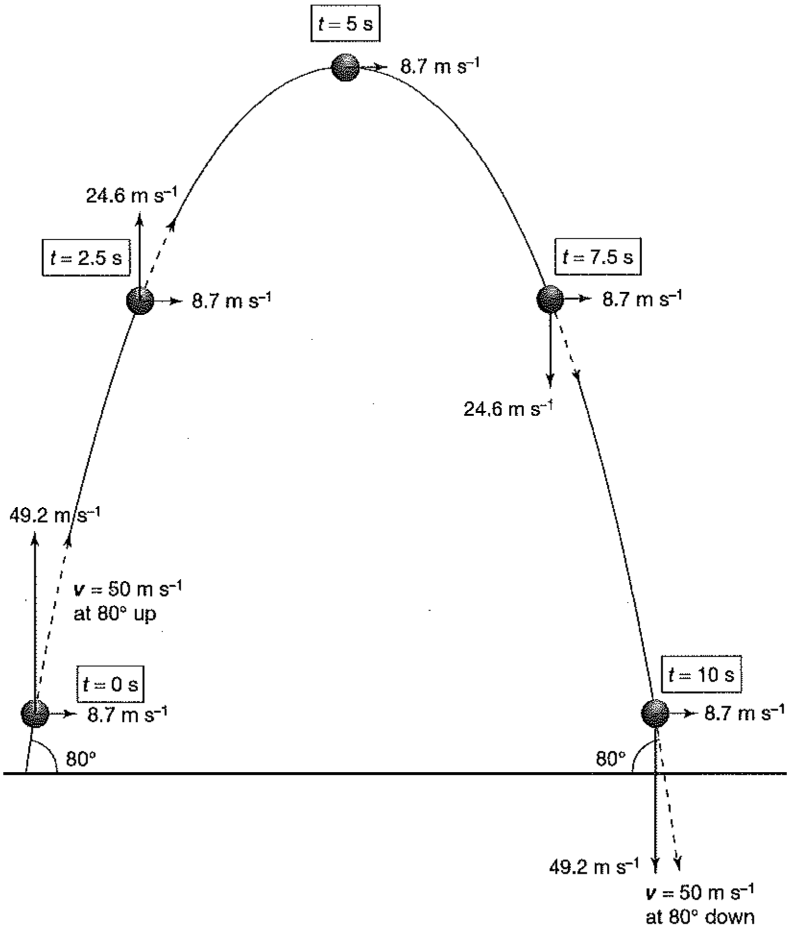


Figure 1.21 Ignoring air resistance, the horizontal velocity of the ball will remain the same, while the vertical component of the velocity will change with time. The motion of the projectile is symmetrical, and for a given height, the ball will have the same speed.

Physics file

It can be shown mathematically that the path of a projectile will be a parabola. Consider a projectile launched horizontally with speed v, and an acceleration down given by g. Let x and y be the horizontal and vertical displacements respectively.

At time t, the horizontal displacement is given by:

$$x = vt \qquad (1)$$

The vertical displacement at time t is:

$$y = \tfrac{1}{2}gt^2 \qquad (2)$$

Substituting (1) into (2) for time, we get:

$$y = \tfrac{1}{2}g\left(\frac{x}{v}\right) = \left(\frac{g}{2v^2}\right)x^2$$

Since g and v do not vary, $y \propto x^2$, which is the relationship for a parabola.

Tips for problems involving projectile motion

- Construct a diagram showing the motion and set the problem out clearly. Distinguish between information supplied for each component of the motion.
- In the *horizontal component*, the velocity, v, of the projectile is constant and so the only formula needed is $v = x/t$.
- For the *vertical component*, the projectile is moving with a constant acceleration (9.8 m s^{-2} down), and so the equations of motion for uniform acceleration must be used.
- In the vertical component, it is important to clearly specify whether up or down is the positive or negative direction, and use this consistently throughout the problem.

Worked example 1.5A

Horizontal launch

A golf ball of mass 150 g is hit horizontally from the top of a 40.0 m high cliff with a speed of 25.0 m s^{-1}. Assuming an acceleration due to gravity of 9.80 m s^{-2} and ignoring air resistance, calculate:

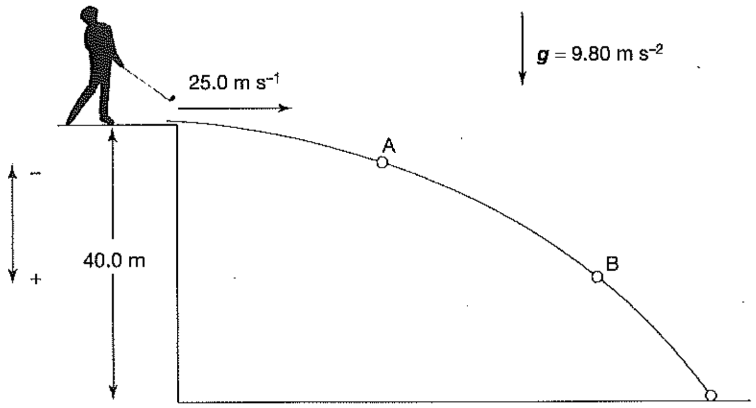
a the time that the ball takes to land

b the distance that the ball travels from the base of the cliff

c the velocity of the ball as it lands

d the net force acting on the ball at points A and B

e the acceleration of the ball at points A and B.



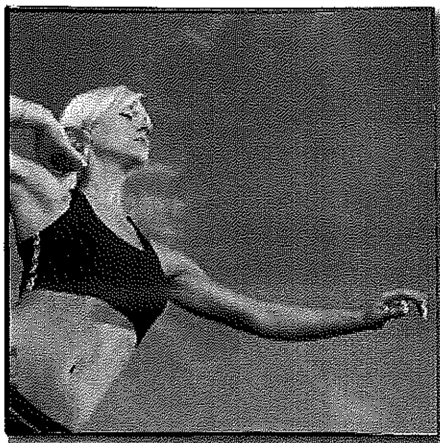


Figure 1.22 Taller athletes have an advantage in the shot-put. They launch the shot from a greater height and so will achieve a greater distance. Recent female world champions have been around 2.0 m tall.

Solution

a To find the time of flight of the ball, you need only consider the vertical component. The instant after it is hit, the ball is travelling only horizontally, so its initial vertical velocity is zero. Taking down as the positive direction:

$\boldsymbol{u}_v = 0, \boldsymbol{a} = 9.80 \text{ m s}^{-2}, \boldsymbol{x} = 40.0 \text{ m}, t = ?$

Substituting in $x = ut + \frac{1}{2}at^2$ for the vertical direction only:

$40.0 = 0 + 0.5 \times 9.80 \times t^2$, so:

$t = \sqrt{40.0/(0.5 \times 9.80)}$

$= 2.86$ s

The ball takes 2.86 s to reach the ground.

b To find the horizontal distance travelled by the ball (i.e. the range of the ball), it is necessary to use the horizontal component.

$\boldsymbol{u}_h = 25.0 \text{ m s}^{-1}, t = 2.86 \text{ s}, \boldsymbol{x}_h = ?$

$x_h = u_h t$

$x_h = 25.0 \times 2.86$

$= 71.5$ m

The ball lands 71.5 m from the base of the cliff (i.e. the range of the ball is 71.5 m).

c To determine the velocity of the ball as it lands, the horizontal and vertical components must be found separately and then added as vectors. From (a), the ball has been airborne for 2.86 s when it lands. The horizontal velocity of the ball is constant at 25.0 m s^{-1}. The vertical component of velocity when the ball lands is:

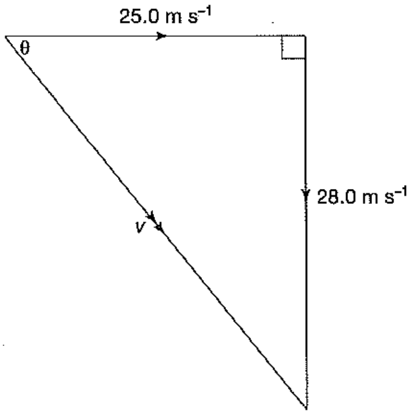
$\boldsymbol{u}_v = 0, \boldsymbol{a} = 9.80 \text{ m s}^{-2}, \boldsymbol{x} = 40.0 \text{ m}, t = 2.86 \text{ s}$

Substituting in $v = u + at$ for the vertical direction only:

$v = 0 + (9.80 \times 2.86)$

$= 28.0 \text{ m s}^{-1}$

The actual velocity, $\boldsymbol{v}$, of the ball is the vector sum of its vertical and horizontal components, as shown in the diagram. The magnitude of the velocity can be found by using Pythagoras's theorem:



$v = \sqrt{25.0^2 + 28.0^2}$

$= \sqrt{1409}$

$= 37.5 \text{ m s}^{-1}$

The angle at which it lands can be found by using trigonometry:

$\tan\theta = \frac{28.0}{25.0}$

$= 1.12$

$\theta = 48.2°$

When the ball hits the ground, it has a speed of 37.5 m s^{-1} and is travelling at an angle of 48.2° below the horizontal.

d If air resistance is ignored, the only force acting on the ball throughout its flight is its weight. Therefore the net force that is acting at point A and point B (and everywhere else!) is:

$\Sigma F = F_g = mg$

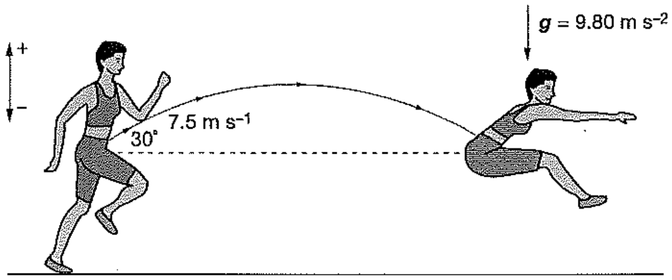
$= 0.150 \times 9.80$

$= 1.47$ N down

e Since the ball is in free-fall, the acceleration of the ball at all points is equal to that determined by gravity, i.e. 9.80 m s^{-2} down.

Worked example 1.5B

Launch at an angle



A 65 kg athlete in a long-jump event leaps with a velocity of 7.50 m s^{-1} at 30.0° to the horizontal. Treating the athlete as a point mass, ignoring air resistance, and using g as 9.80 m s^{-2}, calculate:

a the horizontal component of the initial velocity

b the vertical component of the initial velocity

c the velocity when at the highest point

d the maximum height gained by the athlete

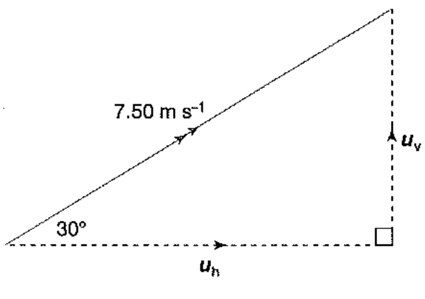
e the total time for which the athlete is in the air

f the horizontal distance travelled by the athlete's centre of mass (assuming that it returns to its original height)

g the athlete's acceleration at the highest point of the jump.

Solution

In this problem, the upward direction will be taken as positive. The horizontal and vertical components of the initial velocity can be found by using trigonometry.



a As shown in the diagram, the horizontal component, u_h, of the athlete's initial velocity is:

$u_h = 7.50 \times \cos 30.0°$

$= 6.50$ m s^{-1} to the right

This remains constant throughout the jump.

b Again referring to the diagram, the vertical component, u_v, of the initial velocity of the athlete is:

$u_v = 7.50 \times \sin 30.0°$

$= 3.75$ m s^{-1} upwards

c At the highest point, the athlete is moving horizontally. The vertical component of the velocity at this point is therefore zero. The actual velocity is given by the horizontal component of the velocity throughout the jump. This was found in (a) to be 6.50 m s^{-1} in the horizontal direction.

d To find the maximum height that is gained, we must work with the vertical component. As explained in (c), at the maximum height the athlete is moving horizontally and so the vertical component of velocity at this point is zero. The vertical displacement of the athlete to the highest point is the maximum height that was reached:

$u_v = 3.75 \text{ m s}^{-1}, v = 0, a = -9.80 \text{ m s}^{-2}, x = ?$

$v^2 = u^2 + 2ax$

$0 = 3.75^2 + (2 \times -9.80 \times x)$

$x = 0.717 \text{ m}$

i.e. the centre of mass of the athlete rises by a maximum height of 72 cm.

e As the motion is symmetrical, the time to complete the jump will be double that taken to reach the maximum height. First, the time to reach the highest point must be found. Using the vertical component:

$u_v = 3.75 \text{ m s}^{-1}, v = 0, a = -9.80 \text{ m s}^{-2}, t = ?$

$v = u + at$

$0 = 3.75 + (-9.80 \times t)$

$t = 0.383 \text{ s}$

The time for the complete flight is double the time to reach maximum height, i.e. total time in the air: $\Sigma t = 2 \times 0.383 = 0.766$ s.

f To find the horizontal distance for the jump, we must work with the horizontal component. From part e, the athlete was in the air for a time of 0.766 s and so:

$t = 0.766 \text{ s}, v = 6.50 \text{ m s}^{-1}, x = ?$

$v = \frac{x}{t}$, so

$x = v \times t$

$= 6.50 \times 0.766$

$= 4.98 \text{ m}$

i.e. the athlete jumps a horizontal distance of 4.98 m.

g At the highest point of the motion, the only force acting on the athlete is that due to gravity (i.e. weight). The acceleration will therefore be 9.80 m s^{-2} down.

Physics file

A conservation of energy approach can also be used for solving projectiles problems. When air resistance can be ignored, the sum of the gravitational potential energy and kinetic energy of the projectile (i.e. its *mechanical* energy) is the same at all points in its flight. At the lowest point in its flight, gravitational potential energy is a minimum and kinetic energy is a maximum. At the highest point, the opposite occurs. The mass needs to be given to determine the actual energy values, but is not required to find the other properties such as acceleration, speed and displacement. Energy will be discussed in detail in Chapter 2.

The effect of air resistance

In throwing events such as the javelin and discus, new records are not accepted if the wind is providing too much assistance to the projectile. In football games, kicking with the wind is generally an advantage to a team; and in cricket, bowling with the wind, across the wind or against the wind can have very different effects on the flight of the ball. The interaction between a projectile and the air can have a significant effect on the motion of the projectile, particularly if the projectile has a large surface area and a relatively low mass.

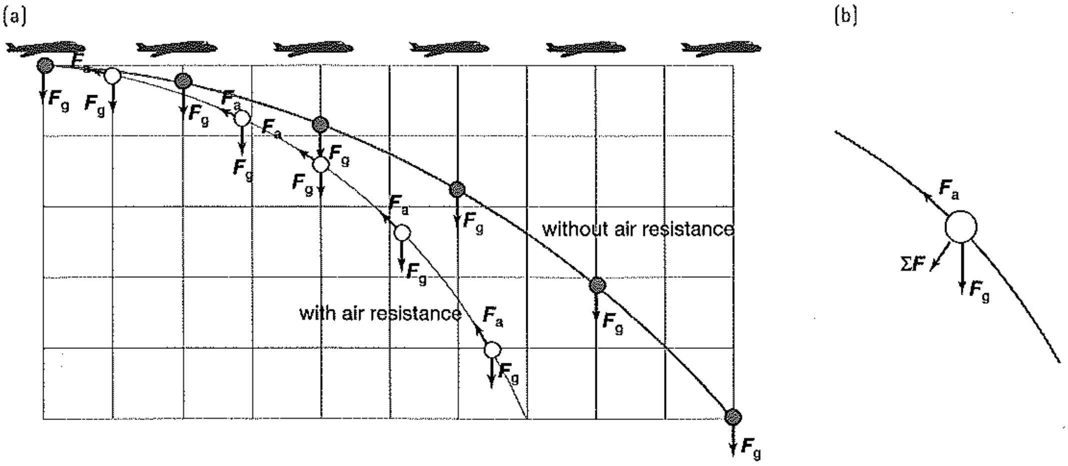


Figure 1.23 (a) The path of a food parcel dropped from a plane. If the plane maintains a constant speed and in the absence of air resistance, the parcel will fall in a parabolic path and remain directly below the plane. Air resistance makes the parcel fall more slowly, over a shorter path. (b) When air resistance is acting, the net force on the parcel is not vertically down.

Figure 1.23 shows a food parcel being dropped from a plane moving at a constant velocity. If air resistance is ignored, the parcel falls in a parabolic arc. It would continue moving horizontally at the same rate as the plane; that is, as the parcel falls it would stay directly beneath the plane until it hits the ground. The effect of air resistance is also shown. Air resistance (or drag) is a retarding force and it acts in a direction that is opposite to the motion of the projectile. If air resistance is taken into account, there are now two forces acting—weight, F_g, and air resistance, F_a. Therefore, the resultant force, ΣF, that acts on the projectile is *not* vertically down. The magnitude of the air resistance force is greater when the speed of the body is greater.

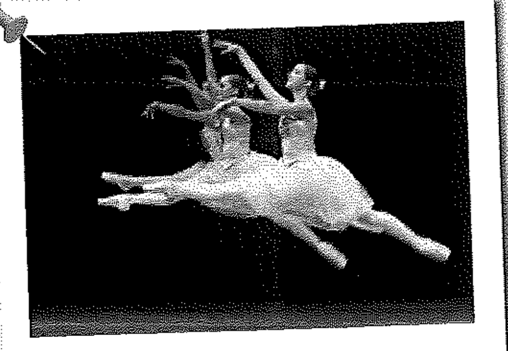
Physics in action

Why ballet dancers seem to float in the air

The grande jeté is a ballet movement in which dancers leap across the stage and appear to float in the air for a period of time. They position their arms and legs to give the impression that they are floating gracefully through the air. The dancer's centre of mass follows a parabolic path. Once the dancer is in mid-air, there is nothing that he or she can do to alter this path.

However, by raising their arms and legs, dancers can raise the position of their centre of mass so that it is higher in the torso. The effect of this is that the dancer's head follows a lower and flatter line than it would have taken if the limbs had not been raised during the leap. The smoother and flatter line taken by the head gives the audience the impression of a graceful floating movement across the stage.

(a)



(b)

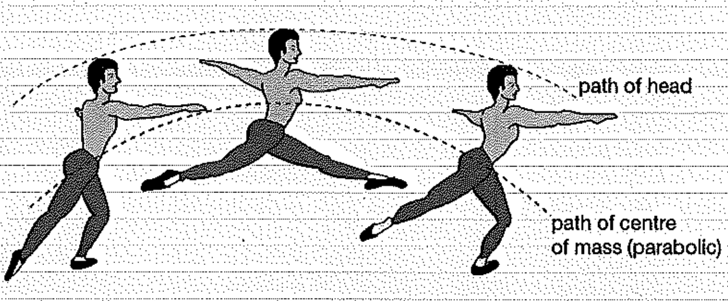


Figure 1.24 (a) Performing the grande jeté. (b) As the position of the centre of mass moves higher in the body, the head of the dancer follows a flatter path and this gives the audience the impression of a graceful floating movement.

1.4 summary

Projectile motion

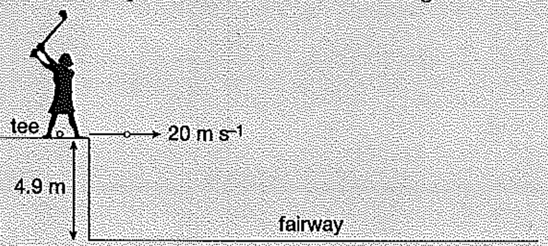
- Projectiles move in parabolic paths that can be analysed by considering the horizontal and vertical components of the motion.
- If air resistance is ignored, the only force acting on a projectile is its weight, i.e. the force of gravity, F_g or W. This results in the projectile having a vertical acceleration of 9.8 m s^{-2} down during its flight.
- The horizontal speed of a projectile remains constant throughout its flight if air resistance is ignored.
- An object initially moving horizontally, but free to fall, will fall at exactly the same rate, and in the same time, as an object falling vertically from the same height.
- At the point of maximum height, a projectile is moving horizontally. Its velocity at this point is given by the horizontal component of its velocity as the vertical component equals zero.
- When air resistance is significant, the net force acting on a projectile will not be vertically down, nor will its acceleration. Under these conditions, the path of the projectile is not parabolic.

1.4 questions

Projectile motion

For the following questions, assume that the acceleration due to gravity is $9.8\ \text{m s}^{-2}$ and ignore the effects of air resistance unless otherwise stated.

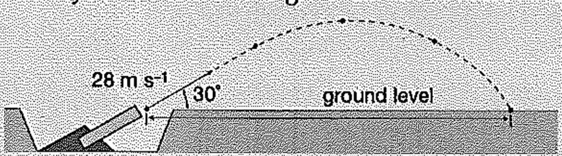
1 A golfer practising on a range with an elevated tee 4.9 m above the fairway is able to strike a ball so that it leaves the club with a horizontal velocity of $20\ \text{m s}^{-1}$.
  a How long after the ball leaves the club will it land on the fairway?
  b What horizontal distance will the ball travel before striking the fairway?
  c What is the acceleration of the ball 0.50 s after being hit?
  d Calculate the speed of the ball 0.80 s after it leaves the club.
  e With what speed will the ball strike the ground?



2 A bowling ball of mass 7.5 kg travelling at $10\ \text{m s}^{-1}$ rolls off a horizontal table 1.0 m high.
  a Calculate the ball's horizontal velocity just as it strikes the floor.
  b What is the vertical velocity of the ball as it strikes the floor?
  c Calculate the velocity of the ball as it reaches the floor.
  d What time interval has elapsed between the ball leaving the table and striking the floor?
  e Calculate the horizontal distance travelled by the ball as it falls.
  f Draw a diagram showing the forces acting on the ball as it falls towards the floor.

The following information applies to questions 3–8.

A senior physics class conducting a research project on projectile motion constructs a device that can launch a cricket ball. The launching device is designed so that the ball can be launched at ground level with an initial velocity of $28\ \text{m s}^{-1}$ at an angle of 30° to the horizontal.



3 Calculate the horizontal component of the velocity of the ball:
  a initially
  b after 1.0 s
  c after 2.0 s.

4 Calculate the vertical component of the velocity of the ball:
  a initially
  b after 1.0 s
  c after 2.0 s.

5 a At what time will the ball reach its maximum height?
  b What is the maximum height that is achieved by the ball?
  c What is the acceleration of the ball at its maximum height?

6 a At which point in its flight will the ball experience its minimum speed?
  b What is the minimum speed of the ball during its flight?
  c At what time does this minimum speed occur?
  d Draw a diagram showing the forces acting on the ball at the maximum height.

7 a At what time after being launched will the ball return to the ground?
  b What is the velocity of the ball as it strikes the ground?
  c Calculate the horizontal range of the ball.

8 If the effects of air resistance were taken into account, which one of the following statements would be correct?
  A The ball would have travelled a greater horizontal distance before striking the ground.
  B The ball would have reached a greater maximum height.
  C The ball's horizontal velocity would have been continually decreasing.

9 A softball of mass 250 g is thrown with an initial velocity of $16\ \text{m s}^{-1}$ at an angle θ to the horizontal. When the ball reaches its maximum height, its kinetic energy is 16 J.
  a What is the maximum height achieved by the ball from its point of release?
  b Calculate the initial vertical velocity of the ball.
  c What is the value of θ?
  d What is the speed of the ball after 1.0 s?
  e What is the displacement of the ball after 1.0 s?
  f How long after the ball is thrown will it return to the ground?
  g Calculate the horizontal distance that the ball will travel during its flight.

10 During training, an aerial skier takes off from a ramp that is inclined at 40.0° to the horizontal and lands in a pool that is 10.0 m below the end of the ramp. If she takes 1.50 s to reach the highest point of her trajectory, calculate:
  a the speed at which she leaves the ramp
  b the maximum height above the end of the ramp that she reaches
  c the time for which she is in mid-air.