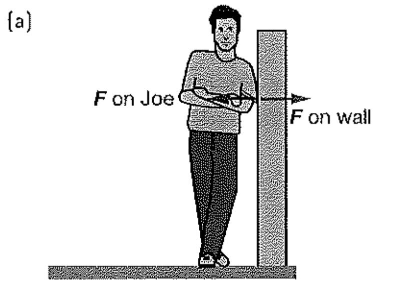
1.3 The normal force and inclined planes

In section 1.2, we reviewed Newton's laws of motion in one dimension. We saw that when unbalanced forces act on an object, it will accelerate. In this section, we will consider some examples of motion in two dimensions. Once again, Newton's laws will be used to analyse these situations.

Normal forces

If you exert a force against a wall, Newton's third law says that the wall will exert an equal but opposite force on you. If you push with greater force, as shown in Figure 1.14, the wall will also exert a greater force. The force from the wall acts at right angles to the surface, i.e. it is *normal* to the surface and is thus called a *normal force*. Like every force, a normal force is one half of an action/reaction pair, so it is often called a *normal reaction force*. In this book, we will use F_N for the normal force although N is also commonly used.



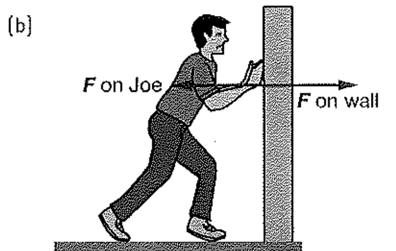


Figure 1.14 (a) If Joe exerts a small force on the wall, the wall will exert a small force on Joe. (b) When Joe pushes hard against the wall, it pushes back just as hard!

A **NORMAL** force, F_N or N, acts at right angles to a surface.

During many interactions and collisions, the size of the normal force *changes*. For example, when a ball bounces, the forces that act on it during its contact with the floor are its weight, F_g or W, and the normal force, F_N, from the floor. As can be seen in Figure 1.15, the normal force is not constant, but changes in magnitude throughout the bounce. When contact has just been made, the ball is compressed only slightly, indicating that the force from the floor is minimal. This force then becomes larger and larger, causing the ball to become more and more deformed. At the point of maximum compression, the normal force is at its maximum value. The normal force from the floor is greatest when the bouncing ball is stationary.

The forces acting on a ball as it bounces (its weight, F_g, and the normal force, F_N) are *not* an action/reaction pair. Both *act on the same body*, whereas Newton's third law describes forces that bodies exert on each other. A pair of action/reaction forces that act during the bounce are the upward force, F_N, that the floor exerts on the ball and the downward force that the ball exerts on the floor. This downward force is equal in magnitude to the normal force, so it varies during the bounce.

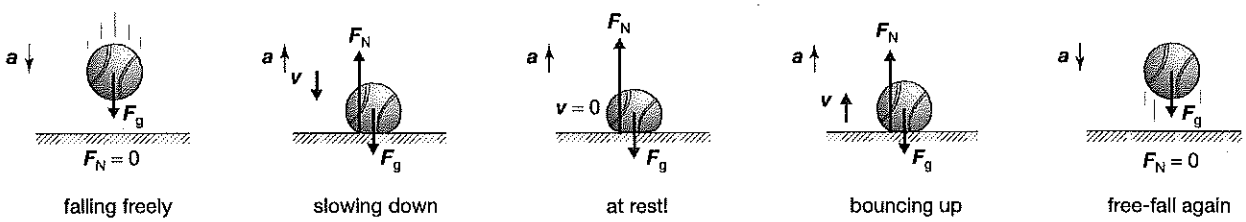


Figure 1.15 The forces acting on a bouncing ball.

Inclined planes

If a toy car is placed on a sloping surface, it will *accelerate* uniformly down the slope. If the angle of the slope is increased, the car will accelerate at a greater rate. To understand this motion, we need to examine the motion of the toy car as it rolls freely across a smooth horizontal surface. The forces acting on the car are its weight, F_g or W, and the normal force, F_N or N, from the surface. If friction is ignored, the car will move with a constant horizontal velocity. The

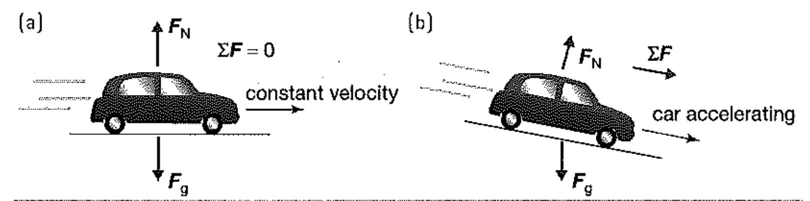


Figure 1.16 The same two forces, F_g and F_N, act on the car when it is (a) rolling horizontally or (b) rolling down an incline. However, these forces are not in balance when the car rolls downhill and so the car accelerates.

car has no motion in the vertical direction, so (as indicated by Newton's first law) the vertical forces must be in balance; that is, $F_N = -F_g$ (Figure 1.16a).

However, if the car now rolls down a smooth plane inclined at an angle θ to the horizontal, the car will *accelerate*. The forces acting on the car are *unbalanced*. If friction is ignored, the only forces acting on the car are still its weight, F_g, and the normal force, F_N, but these forces cannot be balanced because they are not opposite in direction (Figure 1.16b). When the forces are added, they give a net force, ΣF, that is directed down the incline, so the car will accelerate in that direction.

PRACTICAL ACTIVITY 7
Motion on an inclined plane

The usual method of analysing the forces in this situation is to consider the weight, F_g, as having a component that is *parallel* to the incline and a component that is *perpendicular* to the incline (Figure 1.17a). The car is rolling down the incline, so the force parallel to the incline must be responsible for the car's motion. The parallel component of the weight force has a magnitude of $F_g\sin\theta$. Since the car has no motion in the direction perpendicular to the incline, the normal force, F_N, must be equal in magnitude to the perpendicular component of the weight force. This perpendicular component has a magnitude of $F_g\cos\theta$.

If friction is ignored, the parallel component of the weight down the incline is the net force, ΣF, that causes the car to accelerate down the incline. The acceleration, a, of the car down the slope can then be determined from Newton's second law:

$$\Sigma F = F_g \sin\theta$$

so: $ma = mg \sin\theta$

and: $a = g \sin\theta$

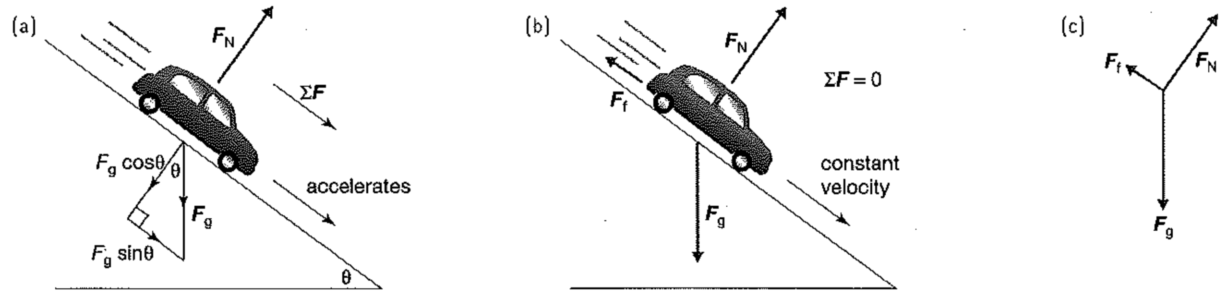


Figure 1.17 (a) The motion of an object on a smooth inclined plane can be analysed by finding the components of the weight force that are parallel and perpendicular to the plane. (b) If friction acts on the car, causing it to move with a constant velocity, the net force on the car will be zero, and so the forces will be in balance both perpendicular and parallel to the incline. (c) A conventional physics diagram of (b) shows the forces acting through the centre of mass.

Acceleration along a smooth incline is given by:

$$a = g\sin\theta$$

What if one of the wheels of the car becomes stuck, so that now a frictional force, F_f, will act? This will be in a direction opposite to the velocity of the car, i.e. up the incline (Figure 1.17b). When the car is moving down the incline with a constant velocity, the forces acting on the car must be in balance. The forces acting parallel to the incline (the frictional force, F_f, and the parallel component of the weight, $F_g \sin\theta$) will therefore be equal in magnitude; that is, $F_f = F_g \sin\theta$.

Physics file

The steepest road in the world is Baldwin St in Dunedin, New Zealand. It has an incline of around 20° at its steepest. This does not sound overly impressive, but angles are deceptive and it is a seriously steep street. In 2000, two university students jumped into a wheelie bin and rolled down Baldwin St. Unfortunately, they crashed into a parked car and suffered serious injuries.

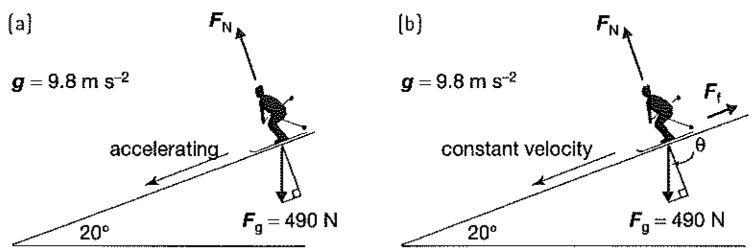
Worked example 1.3A

A skier of mass 50 kg is skiing down an icy slope that is inclined at 20° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is $9.8\ \text{m s}^{-2}$.

a Determine:

- **i** the component of the weight of the skier perpendicular to the slope
- **ii** the component of the weight of the skier parallel to the slope
- **iii** the normal force that acts on the skier
- **iv** the acceleration of the skier down the slope.

b The skier, while on the same slope, runs into a patch of slushy snow that causes her to move with a constant velocity of $2.0\ \text{m s}^{-1}$. Calculate the magnitude of the frictional forces that act here.



skiing over ice — skiing through slush

Solution

The diagrams show the forces acting on a skier who is (a) travelling down an icy slope with negligible friction, and (b) skiing through slushy snow with a constant velocity.

a **i** The weight of the skier $F_g = mg = 50 \times 9.8 = 490$ N down. The perpendicular component of the weight is:

$$F_g \cos\theta = 490\cos 20^\circ = 460\ \text{N}$$

ii The parallel component of the weight is:

$$F_g \sin\theta = 490\sin 20^\circ = 170\ \text{N}$$

iii The normal force is equal to the component of the weight that is perpendicular to the incline, i.e. $F_N = 460$ N.

iv The acceleration of the skier can be determined by considering the forces parallel to the incline. If friction is ignored, the net force acting in this direction is the parallel component of the weight, i.e. 170 N.

$$a = \frac{\Sigma F}{m} = \frac{170}{50} = 3.4\ \text{m s}^{-2}\ \text{down the incline}$$

b If the skier is moving with a constant velocity, the forces acting parallel to the slope must be in balance. In other words, the frictional force is equal in magnitude to the parallel component of the weight force (diagram b), i.e. $F_f = 170$ N.

1.3 summary

The normal force and inclined planes

- A normal force, F_N or N, is the force that a surface exerts on an object that is in contact with it. It acts at right angles to a surface and changes as the force exerted on the surface changes.
- When a ball bounces, the normal force changes throughout the time of contact between the ball and the surface. The normal force is greater than the weight of the bouncing ball for most of the time during their contact, causing the ball to return to the air.
- The normal force, F_N, acting on an object on an inclined plane is equal to the component of the weight force perpendicular to the incline. The steeper the incline, the smaller the normal force.
- For an object that is stationary on a rough inclined plane, the frictional force acts up the incline and is equal in magnitude to the component of the weight force down the slope.
- When an object slides on a smooth inclined plane, its acceleration depends on the angle of the slope:

$$a = g\sin\theta$$

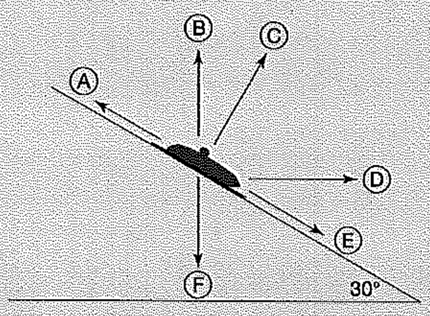
1.3 questions

The normal force and inclined planes

For the following questions, assume that the acceleration due to gravity is $9.8\ \text{m s}^{-2}$ and ignore the effects of air resistance.

The following information applies to questions 1–5.

Kirsty is riding in a bobsled that is sliding down a snow-covered hill with a slope at 30° to the horizontal. The total mass of the sled and Kirsty is 100 kg. Initially the brakes are on and the sled moves down the hill with a constant velocity.



1 Which one of the arrows (A–F) best represents the direction of the frictional force acting on the sled?

2 Which one of the arrows (A–F) best represents the direction of the normal force acting on the sled?

3 Calculate the net frictional force acting on the sled.

4 Kirsty then releases the brakes and the sled accelerates. What is the magnitude of her initial acceleration?

5 Finally, Kirsty rides the bobsled down the same slope but with the brakes off, so friction can be ignored. It now has an extra passenger so that its total mass is now 1400 kg. How will this affect the acceleration of the bobsled?

The following information applies to questions 6–8.

Marshall has a mass of 57 kg and he is riding his 3.0 kg skateboard down a 5.0 m long ramp that is inclined at an angle of 65° to the horizontal. Ignore friction when answering questions 6 and 7.

6 a Calculate the magnitude of the normal force acting on Marshall and his skateboard.
  b What is the net force acting on Marshall and his board?
  c What is the acceleration of Marshall as he travels down the ramp?

7 a If the initial speed of Marshall is zero at the top of the ramp, calculate his final speed as he reaches the bottom of the ramp.
  b Draw a vector diagram of the forces acting on the skateboarder as he moves down the ramp.

8 Marshall now stands halfway up the incline while holding his board in his hands. Calculate the frictional force acting on Marshall while he is standing stationary on the slope.

9 A child rolls a 50 g marble up a playground slide that is inclined at 15° to the horizontal. The slide is 3.5 m long and the marble is launched with a speed of $4.8\ \text{m s}^{-1}$.
  a What is the magnitude of the normal force acting on the marble as it rolls up the slide?
  b How fast is the marble travelling when it is halfway up the slide?

10 A tennis ball bounces on a concrete floor. Discuss the relative sizes of the weight of the ball, F_g, and the normal force, F_N, that the floor exerts on the ball when:
  a the ball is in mid-air falling towards the floor
  b the ball has just made contact with the floor and is slowing down
  c the ball is at the point of its maximum compression.